

FUN DRAWING WITH MATHEMATICS (LEVEL 3)

Description	Learners use Mathematics to develop some drawing techniques, and then use their products to create puzzles to entertain the family.
Leading Question	How can Mathematics awaken the artist within me and help me improve my drawing skills?
Total Time Required	~5 hours over 3 days
Supplies Required	Paper and pencil, (optional: removable stickers like sticky notes).
Subjects	Mathematics, Art and Design
Supervision	Low
Learning Outcomes	By the end of the learning, the learner will be able to: <ol style="list-style-type: none"> 1. Use grids for graphing and estimating areas 2. Practice geometric reflection and simple rotation of 2-D shapes 3. Construct complex drawings using grids and transformations
Previous Learning	<ul style="list-style-type: none"> ● 2-dimensional shapes and angles (surfaces, areas) ● Locating points on a coordinate grid

DAY 1

Today you will learn how to use drawing and mathematics together.

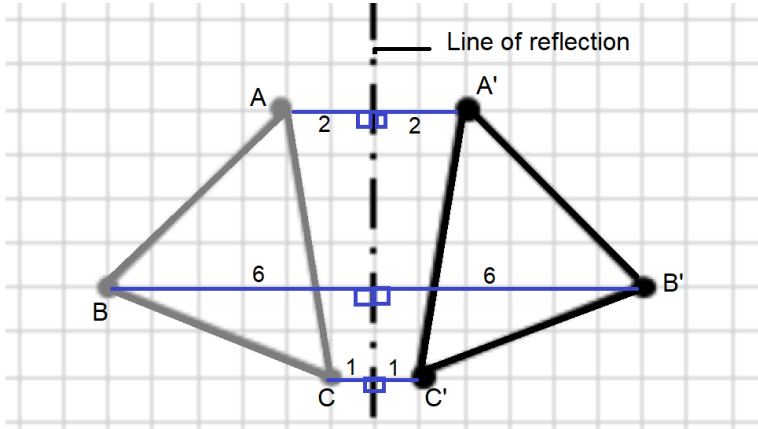
Suggested Duration	Activity and Description
5 minutes	<ul style="list-style-type: none"> ● Introduce the project's leading question: How can Mathematics awaken the artist within you and help you improve your drawing skills? ● And ask: have you ever used Mathematics in drawing? How?
30 minutes	<ul style="list-style-type: none"> ● Let's start by reviewing the grid graph, and using ordered pairs (x,y) to represent points on a grid. ● Math talk: <ul style="list-style-type: none"> ○ What is the meaning of the expression "(x,y)"? ○ Can you think of a different way to explain it? ○ How does this relate to drawing? ● In this pair:

	<p>“x”, or the first number in the ordered pair, represents how many steps across (horizontally, or sideways) we move from the Origin point;</p> <p>“y”, or the second number, represents how many steps we go upwards or downwards (vertically).</p> <p>Therefore, this ordered pair (x,y) tells us the location of a certain point on a grid.</p> <ul style="list-style-type: none"> ● Solve question 1 on day 1 worksheet.
30 minutes	<p>Math Talk”</p> <ul style="list-style-type: none"> - Can you recall some 2D shapes? - Have you ever made any calculations with those shapes? Can you give an example of one? (i.e. counting the number of sides, counting the length of the sides, counting the number of squares that can fit into one shape, etc.) <ul style="list-style-type: none"> ● An <i>area</i> is one calculation that we can do with 2D shapes. It measures how much surface a shape occupies. This can be easily found on a grid by counting the number of unit squares that are inside the shape. Some shapes may have half squares, or smaller parts of squares, so you can use some estimation to find their area approximately. ● Solve question 2 on day 1 worksheet.
25 minutes	<p>In real life, not all shapes are rectangles or triangles. To calculate areas of irregular shapes, one method is to divide the shapes into regular parts.</p> <p>Follow the instructions in question 3 on the day 1 worksheet and see how you can find the area of such an irregular shape using this method.</p> <p>Present and explain your solutions to a parent/instructor or older sibling/ peer and engage in a conversation around these questions:</p> <ul style="list-style-type: none"> - What process did you use to find the answer in each case? - How is this process like others that you have used? - Have you ever solved problems like this before? Maybe in other contexts? Can you give me an example?

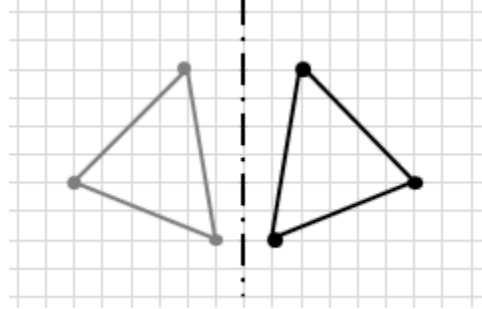
DAY 2

Today, we will practice Geometric Transformations. A geometric transformation is a change of shape.

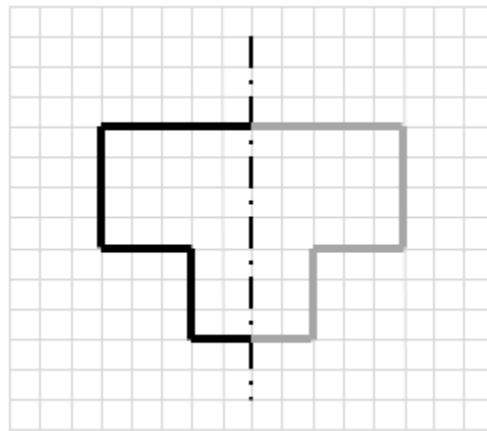
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Suggested Duration	Activity and Description
5 minutes	<ul style="list-style-type: none"> ● A transformation can change a shape's position, orientation and its size. <ul style="list-style-type: none"> ○ Can you give me some examples of this? ● There are four types of geometric transformations: translation, reflection, rotation and enlargement. Each transformation follows certain rules. ● Today, you will learn to do 2 types of geometric transformations: Reflection and Rotation.
10 minutes	<p>Reflection</p> <p>A reflection is a transformation that flips a shape or line around a given line called the line of reflection or axis of reflection</p> <p>1.</p> <p>An example of a reflection</p>  <p>A'B'C' is a reflection of ABC.</p> <ul style="list-style-type: none"> ● Point A' is a reflection of point A. They are both 2 squares away from the line of reflection ● Point B' is a reflection of point B. They are both 6 squares away from the line of reflection

- Point C' is a reflection of point C . They are both 1 square away from the line of reflection



The figure below is symmetrical, so reflecting one half will complete the figure:



Based on these examples, try to find some patterns (i.e. generalize) and list some properties of a geometric reflection. Here are some properties, make sure to encourage learners to say the same in different ways:

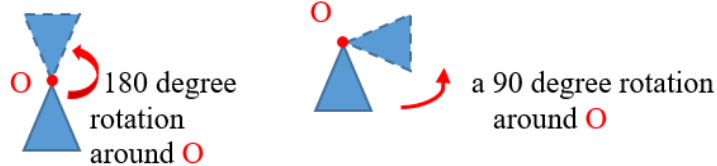
- If you were to fold the diagram along the line of reflection, the reflected shape would exactly fit over the original shape.
- Each point in the shape is the same perpendicular distance from the line of reflection as the corresponding point in the object
- The reflected shape is the same size as the original shape

30 minutes

- In the Day 2 worksheet, complete the reflections in Question 1 and Question 2, share and explain your solutions to a parent/instructor or older peer.
- Revise your solutions based on the feedback
- Assessment criteria:

	<ul style="list-style-type: none"> <input type="checkbox"/> The learner uses a mix of imagination, and physical means to verify responses. (by placing the sheet against an actual mirror to see how the reflected shape looks) <input type="checkbox"/> The ability to explain or defend their answers
<p>20 minutes</p>	<p>Rotation</p> <ul style="list-style-type: none"> ● Another kind of transformation is Rotation. A geometrical rotation is a transformation that turns a figure or shape around a given point called the centre of rotation. ● Let's start with a simple activity. <ul style="list-style-type: none"> ● Stand up facing one side of the room. ● While standing in the same position, make one full rotation to go back to the initial position. ● What angle have you covered in this rotation? (it is 360 degrees, or a full circle). ● Now make a 180-degree rotation. This is a half-circle; it makes you face the opposite direction in the room. ● How about a 90-degree rotation? ● If you are facing East (where the sun rises), a 90-degree rotation can make you either facing North or South. ● (Optional: In order to make a general agreement, mathematicians agree that a rotation is usually done counterclockwise, that is opposite to the clock rotation movement. So, if you are facing east, a rotation of 90 degrees will make you face North. Rotating from East to South is a 270-degree rotation). ● An example of Rotation is the difference between when a door is closed versus when it is open <div data-bbox="477 1499 928 1724" data-label="Image"> </div> <ul style="list-style-type: none"> ● A rotation is defined by a center of rotation and an angle

- Stand facing the wall in the room and rotate by 90 degrees. Where are you facing now?
- See the diagram below 2 examples of rotation: The center of rotation must be defined.



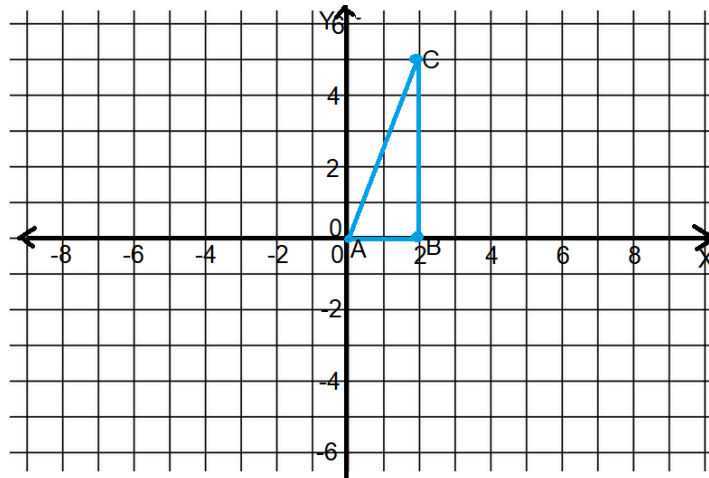
Properties of a geometric rotation

The object and the image of a rotated object have some mathematical properties

1. A rotation maps a line to a line, a segment to a segment and an angle to an angle
2. A rotation preserves the lengths of segments
3. A rotation preserves the measurement of angles

Example of a rotation.

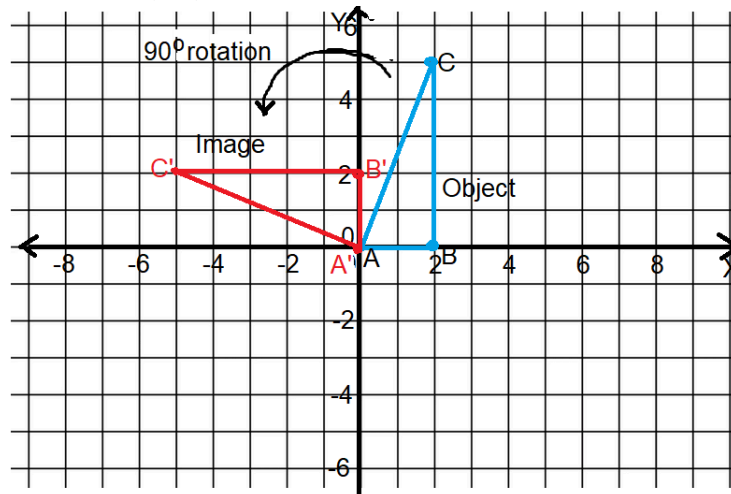
Triangle ABC has the following vertices $A(0,0)$, $B(2,0)$ and $C(2,5)$



- (i) Rotate the triangle ABC 90° counterclockwise about the origin $(0,0)$. Draw the image $A'B'C'$ of the rotated triangle on the grid. What are the vertices of the image $A'B'C'$?
- (ii) Rotate the triangle ABC 90° clockwise about the origin $(0,0)$. Draw the image $A''B''C''$ of the rotated triangle on the grid. What are the vertices of the image $A'B'C'$?

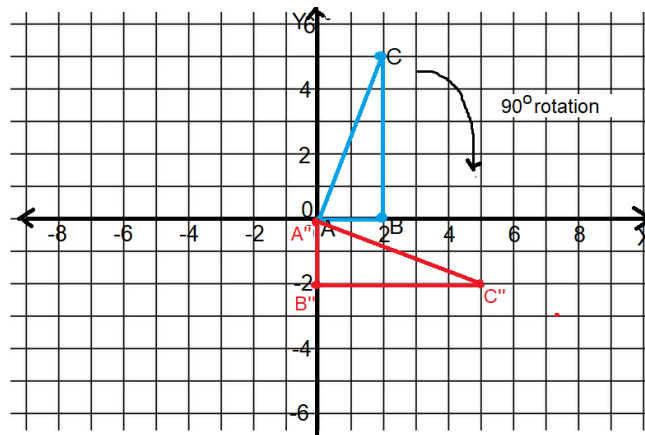
Solution

- a. Triangle ABC rotated 90° counterclockwise about the origin point $(0,0)$ as the centre of rotation



The image has the following vertices $A'(0,0)$, $B'(0,2)$ and $C'(-5,2)$

- (i) Triangle ABC rotated 90° clockwise about the origin point $(0,0)$ as the centre of rotation



The image $A''B''C''$ has the following vertices $A''(0,0)$, $B''(0,-2)$ and $C''(5,-2)$

20 minutes

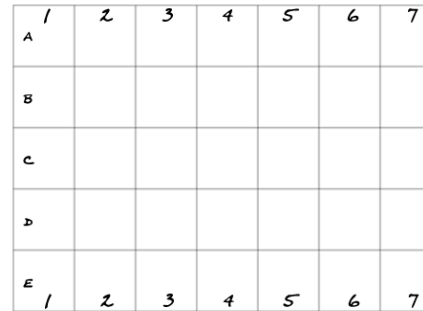
- Solve day 3 and 4 on the day 2 worksheet, and then explain your solution to a parent or adult.
- Assessment criteria:

	<ul style="list-style-type: none"> <input type="checkbox"/> The learner uses a mix of imagination, and physical means to verify responses (by putting the pencil on the Origin of rotation, and turn the whole sheet around the origin shows where the rotated shape will be) <input type="checkbox"/> Depth of understanding through the confidence in explaining and defending their answer
15 minutes	<p>Reflection</p> <ul style="list-style-type: none"> ● Learners are asked reflection questions: <ol style="list-style-type: none"> 1. Describe Reflection in your own words 2. What does Reflection change in an object? (size, color) 3. Describe Rotation in your own words 4. What does Rotation change in an object? (colors size, weight) 5. What 3 important things have you learned about reflection 6. What 3 important things have you learned about rotation? 7. What else would you like to learn about reflection and rotation?

DAY 3

Today you will use grids and reflections to make your drawing skills better! .

Suggested Duration	Activity and Description
10 minutes	<p>Grid method of drawing</p> <ul style="list-style-type: none"> ● When drawing anything, you need to get the proportion right. One way of achieving this is to aid your drawing with the use of a grid method of drawing. ● The grid method of drawing involves placing a grid of squares over a reference photo and placing lightly an identical grid of squares on your drawing paper ● Example



Source: <https://www.art-is-fun.com/grid-method>

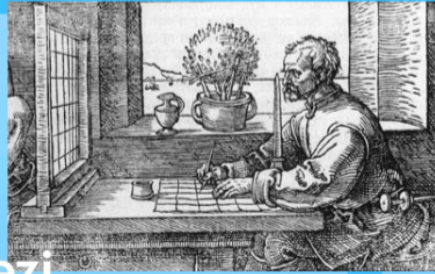
Remember when drawing the grids, the grids must have a 1:1 ratio. A 1:1 ratio means that you will have the exact same number of lines on your paper as you have on the reference photo and that the grid lines must be equally spaced – perfect squares.

You then draw your subject on one square at a time and replicate what you see in each square

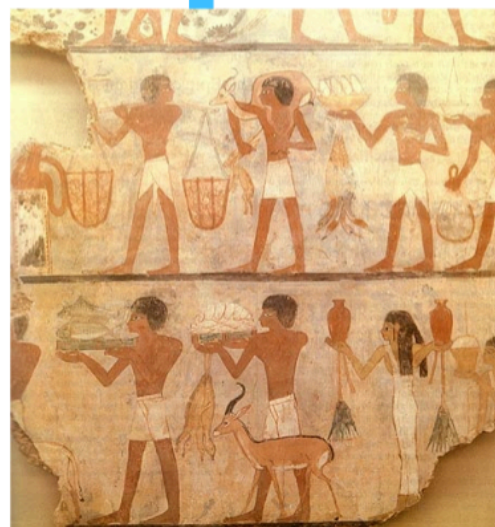
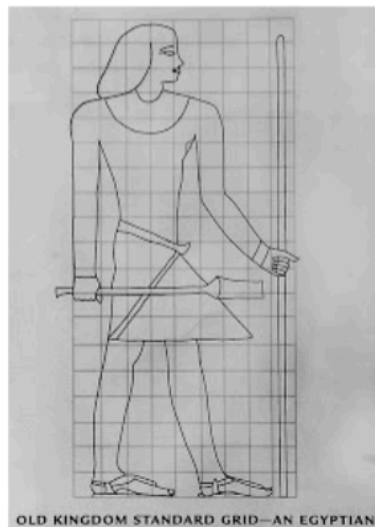
Once you're finished, you simply erase or paint over the grid lines on your paper and start working on your painting which will now be in perfect proportion

- Many artists have used grids in drawing throughout history, see the below examples from ancient Egyptians (more than 4000 years ago), or Renaissance artists (more than 500 years ago).

Renaissance artists would build a wooden frame, hammer in nails spaced equally apart and tie lengths of string from one end to the other to create their rows and columns. They would then place the wooden frame where they could look through the grid and see their subject from their workspace.



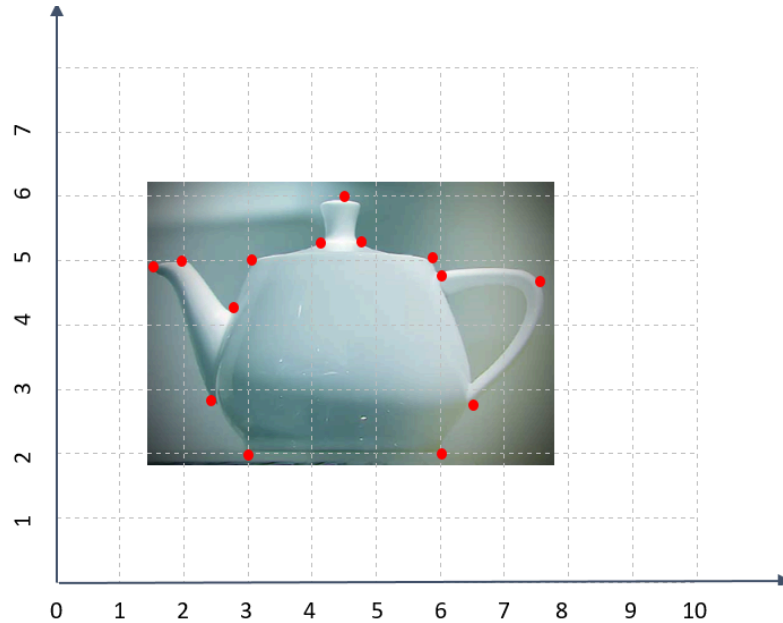
The ancient Egyptians at the beginning of the Middle Kingdom (around 2055 BC - 1650 BC) used grids by taping a string soaked in red dye against their drawing surface to create grid lines. Their figures had very specific proportions that didn't change for centuries!



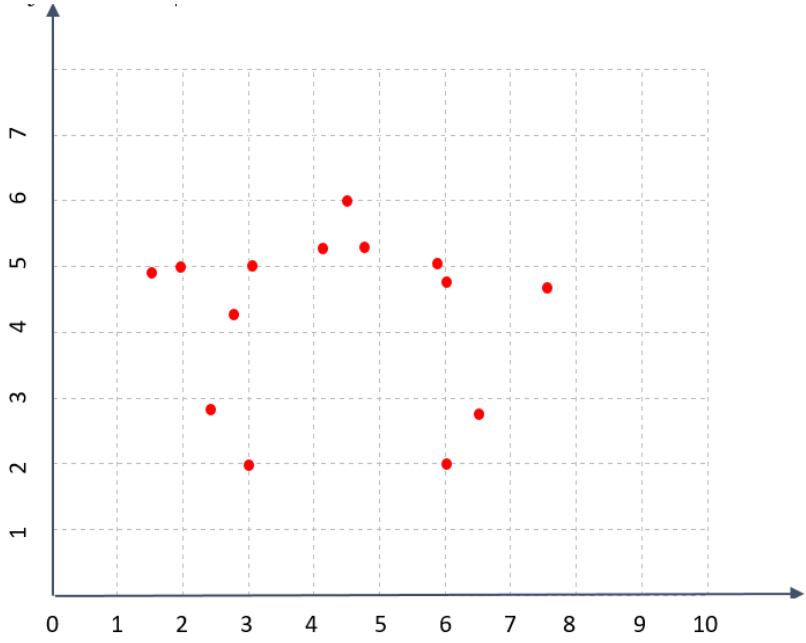
Source: <https://prezi.com/eijrwmwhvgrg/grid-drawing-for-6th-grade/>

30 minutes

- Let's start with an example: see this image of a teapot (called the UTAH teapot)
- First try to draw this teapot on a piece of paper. (The learner takes 5-10 minutes to draw the teapot).
- Now let's use the grid technique and try again.
 - Place the teapot through a grid and mark the main points of the shape (see demonstration below).



- Now try to draw the pot again using the dots.

	
5 minutes	<ul style="list-style-type: none"> ● Reflect on your own learning. Questions to reflect on: <ol style="list-style-type: none"> 1. To what extent has the grid technique improved your drawing? 2. Do you think it is easier to draw using a grid? 3. What objects would you like to try drawing using this grid technique?
20 minutes	<ul style="list-style-type: none"> ● Solve problem 1 on the Day 3 Worksheet
20 minutes	<ul style="list-style-type: none"> ● Solve problem 2 on the Day 3 Worksheet
30 minutes	<ul style="list-style-type: none"> ● There are some objects that we call 'Symmetrical.' This means that one half of the object is a mirror image of the other half. ● Is a circle symmetrical? What about a square? How about the human body? ● Try to use the grid technique and Reflections to draw the ladybird image in Question 3 of Day 3 worksheet.
5 minutes	<ul style="list-style-type: none"> ● Hang the drawings you completed today on a wall in the house.

Additional enrichment activities	<ul style="list-style-type: none"> ● Make another copy of the ladybird drawing, and cut it into square pieces, then challenge your family members to arrange the puzzle. ● Learners are challenged to choose a symmetric shape from the house, trace it into a grid, and make 2 copies of it. Then cut the 2 drawings into smaller square puzzle pieces and arrange a competition amongst 2 groups of family members to complete the puzzle in a shorter period of time. ● Learners use a grid to draw an image or an object of their choice. Once done, they may display it on a wall in the house.
Modifications for simplification	<ul style="list-style-type: none"> ● For a simpler version, learners may skip Question 2 on day 2 and problem 3 of day 3.

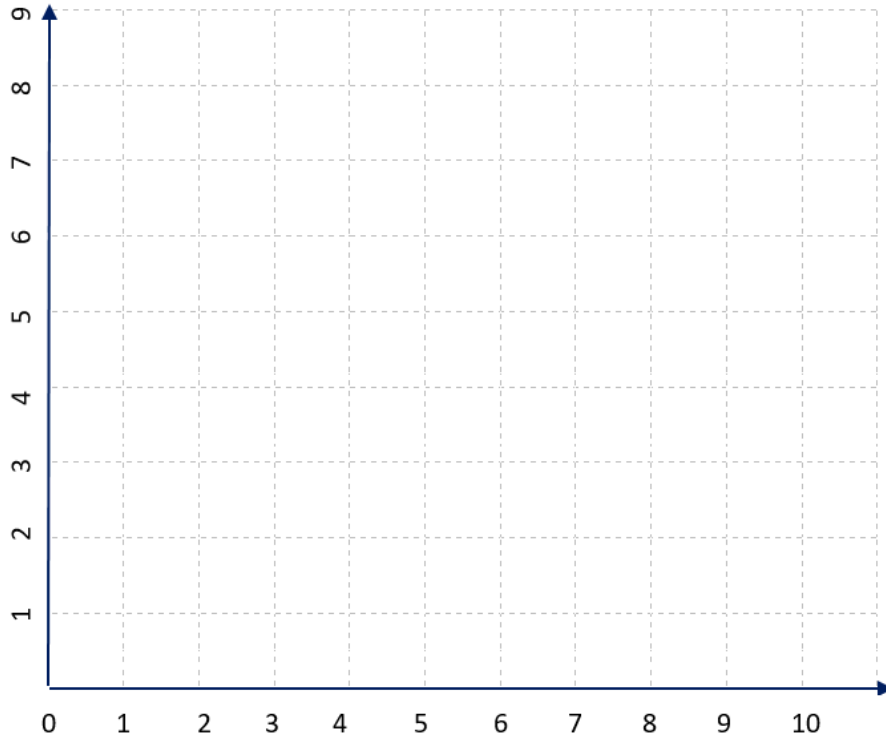
ASSESSMENT CRITERIA

By the end of the learning, the learner should be able to point out:

- The accuracy of drawings compared to the original images.
- The rate of progress in terms of becoming faster at using the techniques.
- Use of properties of geometrical reflections in drawing reflection images
- Use of properties of geometrical reflections in drawing reflection images
- Accurate location of points on a coordinate grid

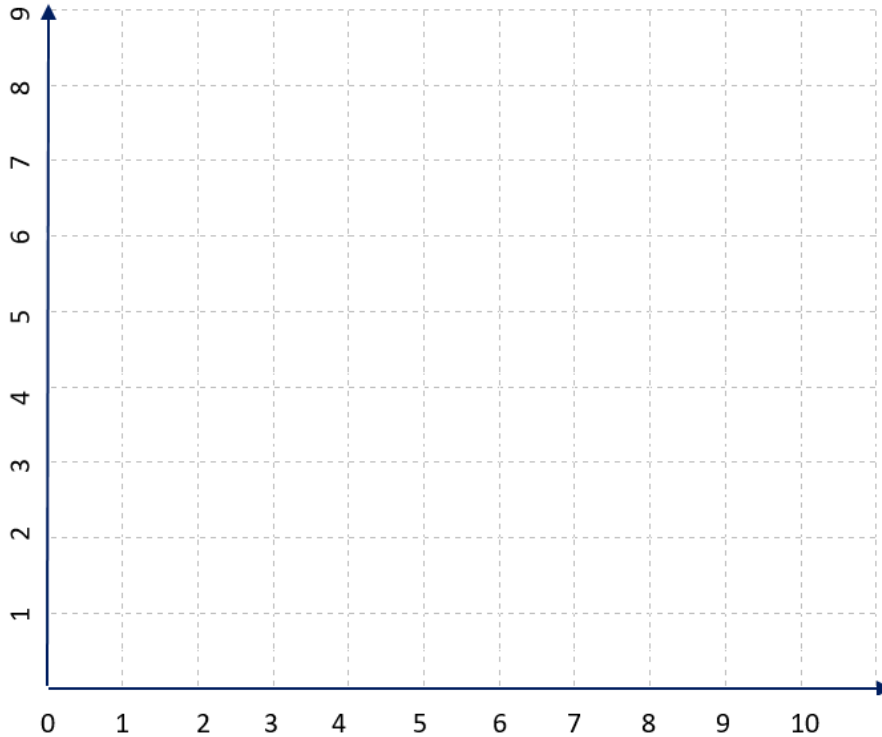
DAY 1 WORKSHEET

1. On the grid below, plot the points A(1,1), B(4,1), C(4,4) and D(1,4).



- a. What is the shape ABCD? Connect the points A to B, B to C, C to D, and D to A.
 - b. Plot the points E(2,5), F(4,5), G(4,7). Connect the points E to F, F to G, and G to E. What is the shape EFG?
 - c. Plot the points H(5,5), I(10,5), & J(10,7). Connect the dots H to I, I to J, and J to I. What is the shape HIJ?
2. a. Find the area of the shape ABCD, by counting the number of area units, or grid squares, it encloses.
- b. Find the area of shape EFG. (hint: the answer is a whole number).
- c. Can you find the area of shape HIJ?
- Probably it will be easier if you add a point K(5,7). HIJK is a rectangle.
 - First, count the number of area units inside HIJK;
 - Then the area of HIJ is half of that.

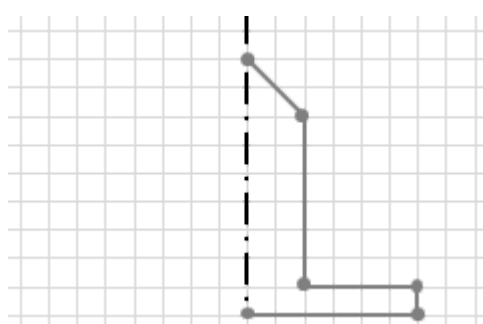
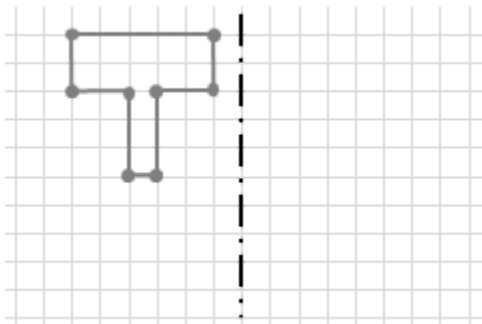
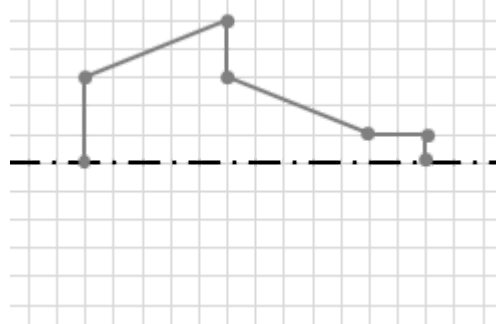
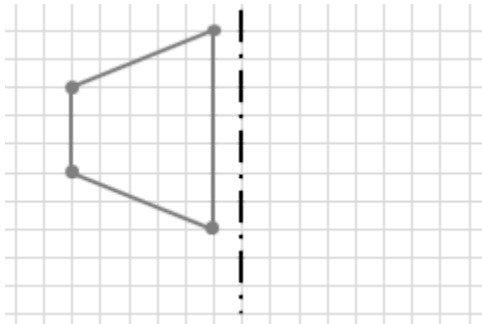
3. On the grid below

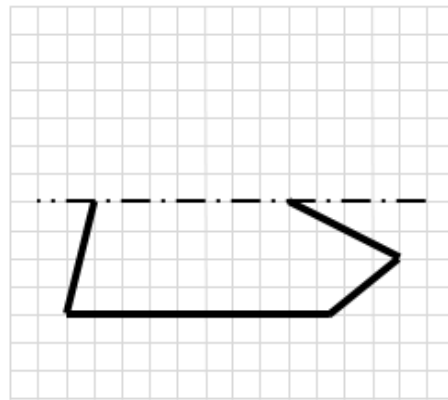
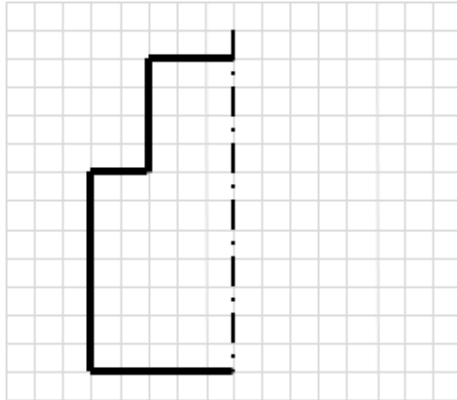


- Draw the points A(3,2), B(9,2), C(9,7), D(7,5) and E(3,9).
- Connect the points: A to B, B to C, C to D, D to E, and E to A.
- Can you calculate the area of shape ABCDE?
 - Hint: Add two points: F(9,5), and G(3,5). The area of ABCDE can be found by adding the areas of 3 shapes:
 - Area of rectangle ACFG + Area of triangle CDF + Area of triangle DEG.

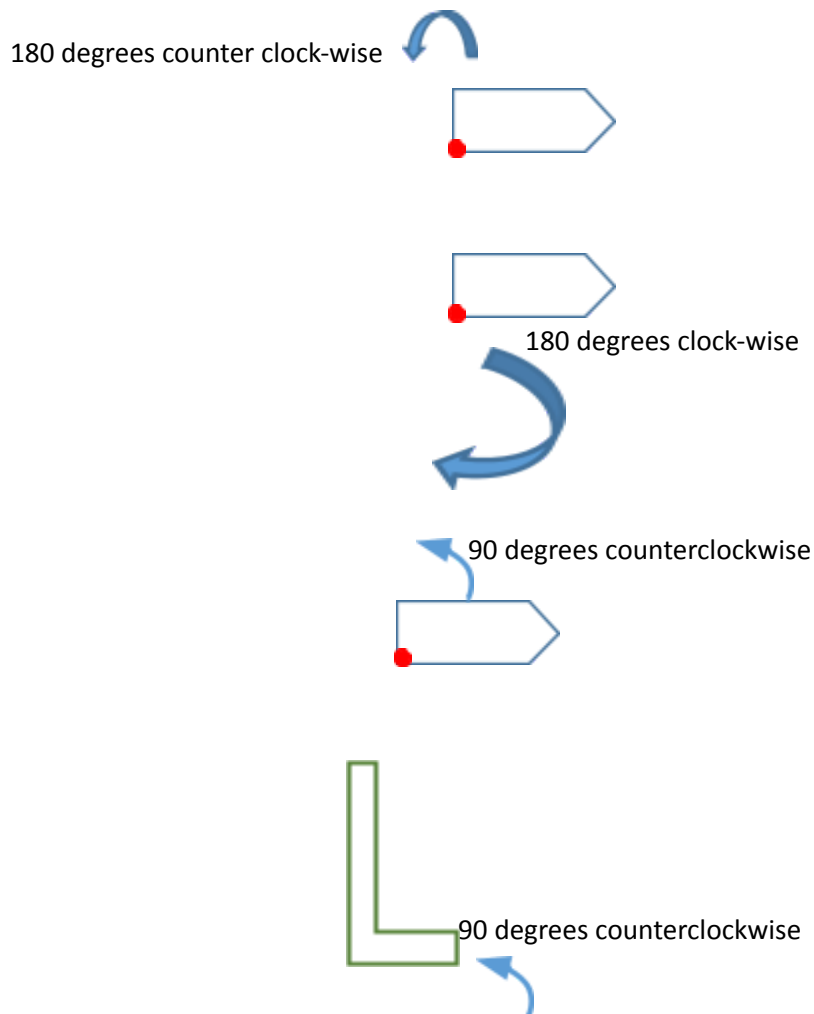
DAY 2 WORKSHEET

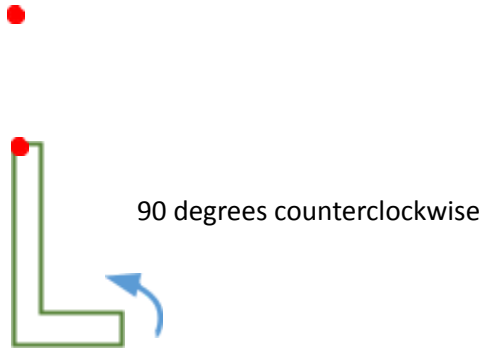
1. Draw the reflection of the shapes using the dashed line as Axis of Reflection (source: <https://www.helpingwithmath.com/printables/worksheets/geometry/4g3-symmetry02.htm>)
Hint: The reflection of each point will be the same number of squares to the other side of the mirror line.





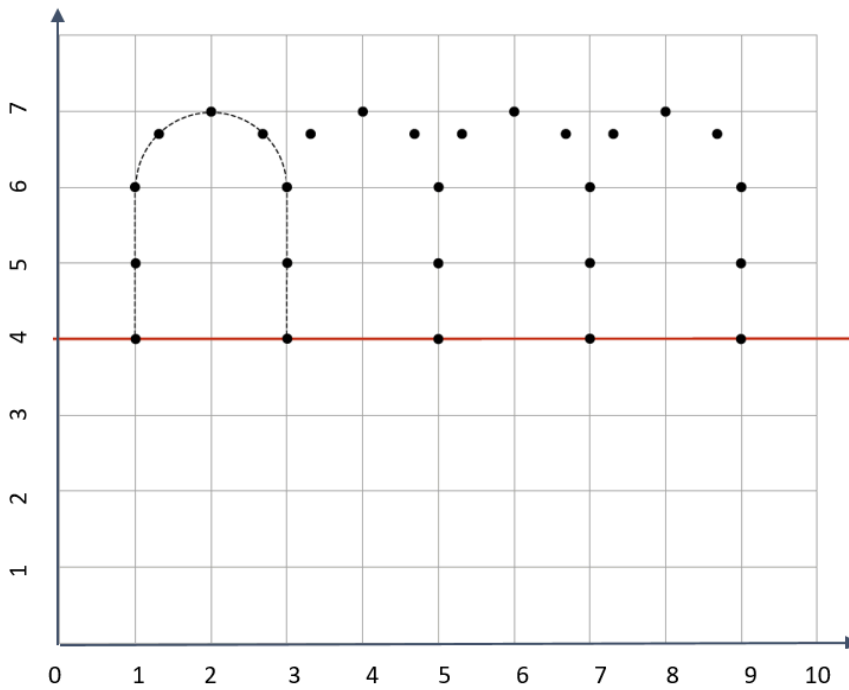
2. Draw the rotated shape around the origin (red dot), and with the respective angle:





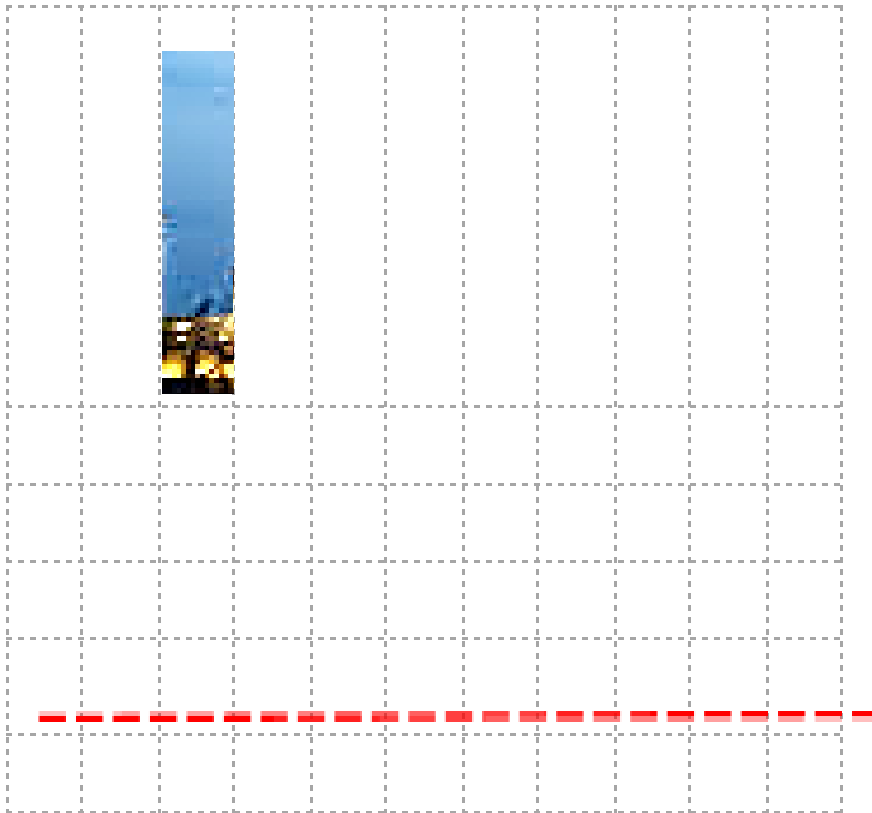
DAY 3 WORKSHEET

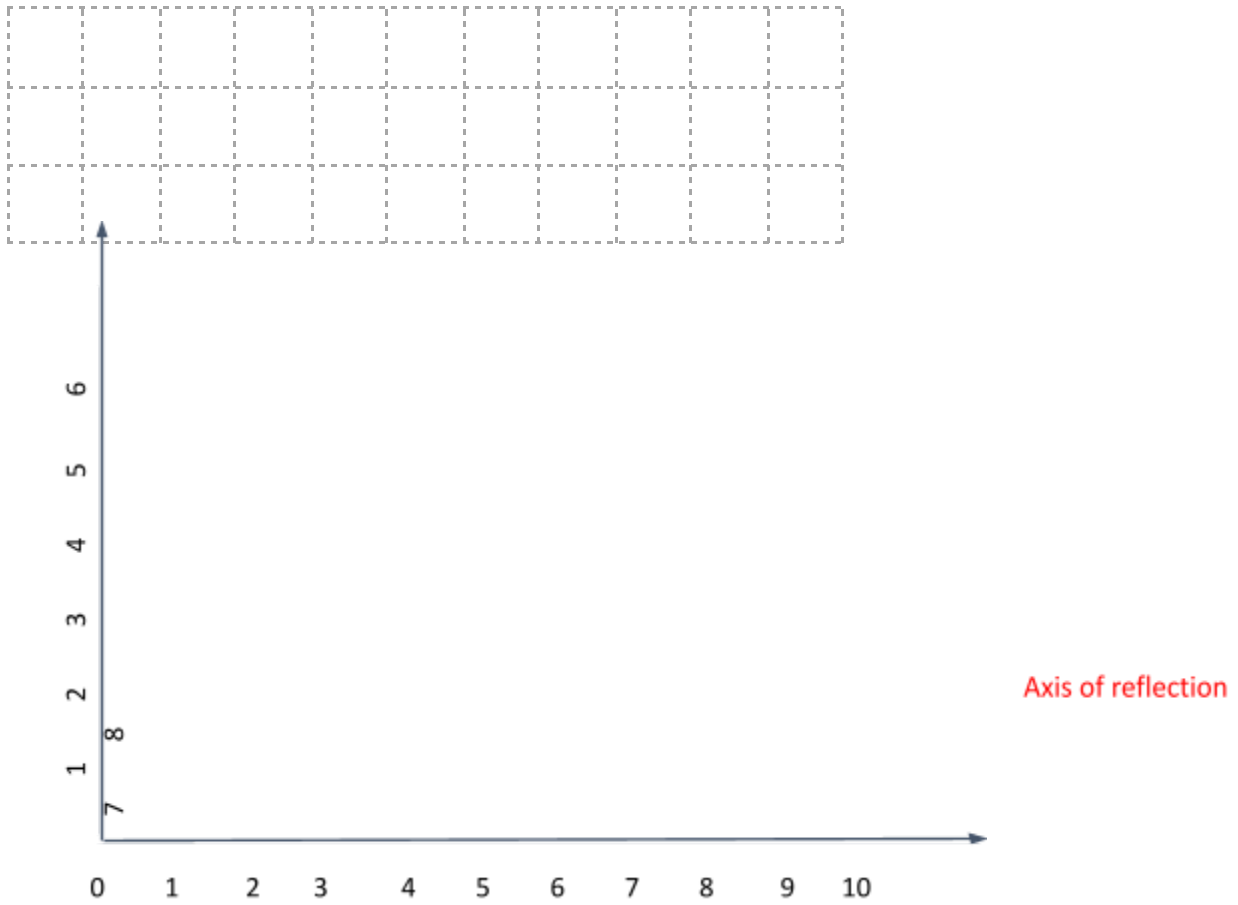
1. Here is a picture from the backyard of the Museum of Islamic Art in Qatar.
 - a. Using the grid and points below, try to trace the arches.



The mirror line, or
Axis of Reflection

- b. After drawing the arches, use the red line as an axis of reflection, and draw the reflection of the arches.
2. Here is another picture of the Museum of Islamic Art with its reflection in the water. Use the grid below to draw the reflection of the museum.





Hint: first mark the main corner points of the museum structure, then make the reflection of each point, and lastly connect them to get the trace of the museum building's reflection.

3. Notice the ladybird image below. You are challenged to recreate this image on the grid below.
Hint: The Ladybird's body is symmetrical, which means that one side of it is a reflection of the other side- see the dashed line below is the axis of symmetry.

