Fun drawing with mathematics (Level 3)

Ages 11 to 14 (Level 3)

Description:	Learners use Math to develop some drawing techniques, and then use their products to create puzzles to entertain the family.
Leading question:	How can Math awaken the artist within me and help me improve my drawing skills?
Age group:	11-14
Subjects:	Mathematics: Shapes, measurements and patterns Art: Math based art using shapes and patterns
Total time required:	~ 5 hours over 3 days
Self-guided / Supervised activity:	Low parent supervision required
Resources required:	Paper and pencil, (optional: removable stickers like sticky notes).

Learning outcomes:	- Using grids for graphing and estimating areas - Practicing geometric reflection and simple rotation of 2-d shapes - Constructing complex drawings using grids and transformations
Required previous learning:	- 2 dimensional shapes (surfaces, areas) - Angles - Locating points on a coordinate grid
Inspiration:	Grid drawing for 6th graders https://prezi.com/eijrwmwhvgrg/grid-drawing-for-6th-grade/

Topics/concepts covered and skills developed
• Coordinate grid • Geometrical reflection • Geometrical rotation • Grid method of drawing • Communication skills • Drawing

Day	Time	Activity and Description
1	5 mins	Introduce the project's leading question: How can Math awaken the artist within you and help you improve your drawing skills? And ask: have you ever used Maths in drawing? How?
	30 mins	Let's start by reviewing the grid graph, and using ordered pairs (x,y) to represent points on a grid. Invite learners to draw a grid graph (a vertical and a horizontal line intersecting in a perpendicular way) and draw a point anywhere on the grid. That point will be the pair (x, y). **Math Talk**: Ask learners: - What is the meaning of the expression "(x,y)"? - Can you think of a different way to explain it? - How does this relate to drawing? **Hint**: In this pair, "x", or the first number in the ordered pair, represents how many steps across (horizontally or sideways) we move from the Origin point;"y", or the second number, represents how many steps we go upwards or downwards (vertically). Therefore, this ordered pair (x,y) tells us the location of a certain point on a grid. Solve question 1 on day 1 worksheet.
	30 mins	Mah Talk- Ask the learners: - Can you recall some 2D shapes? - Have you ever made any calculations with those shapes? Can you give an example of one? (i.e. counting the number of sides, counting the length of the sides, counting the number of squares that can fit into one shape, etc.) An *area* is one calculation that we can do with 2D shapes. It measures how much surface a shape occupies. This can be easily found on a grid by counting the number of unit squares that are inside the shape. Some shapes may have half squares, or smaller parts of squares, so you can use some estimation to find their area approximately. Solve question 2 on day 1 worksheet.
	25 mins	In real life, not all shapes are rectangles or triangles. To calculate areas of irregular shapes, one method is to divide the shapes into regular parts.

		Follow the instructions in question 3 on the day 1 worksheet and see how you can find the area of such an irregular shape using this method. Learners will present and explain their solutions to a parent, an older sibling or a peer and engage in a conversation around these questions: - What process did you use to find the answer in each case? - How is this process like others that you have used? - Have you ever solved problems like this before? Maybe in other contexts? Can you give me an example?
2	5 mins	Today, we will practice Geometric Transformations. A geometric transformation is a change of shape. A transformation can change a shape's position, orientation and its size. Ask the learners: can you give me some examples of this? There are four types of geometric transformations: translation, reflection, rotation and enlargement. Each transformation follows certain rules. Today, you will learn to do 2 types of geometric transformations: Reflection and Rotation.

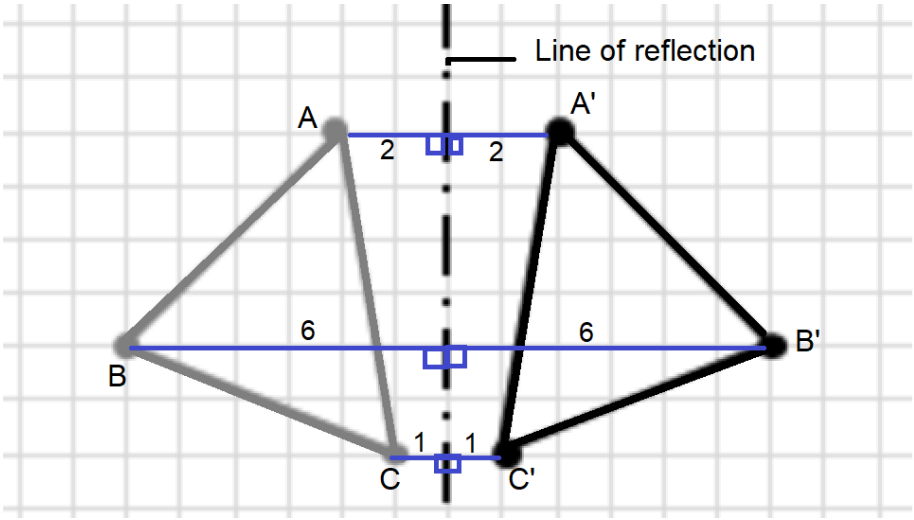
10 mins

Reflection

A reflection is a transformation that flips a shape or line around a **given line** called the line of reflection or axis of reflection

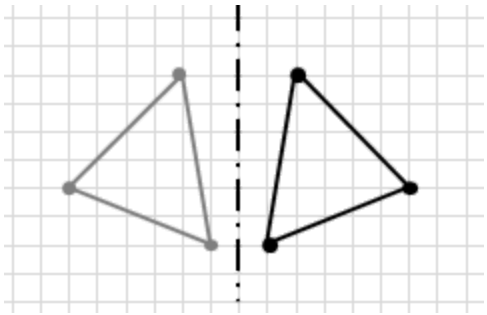
1.

Example of a reflection

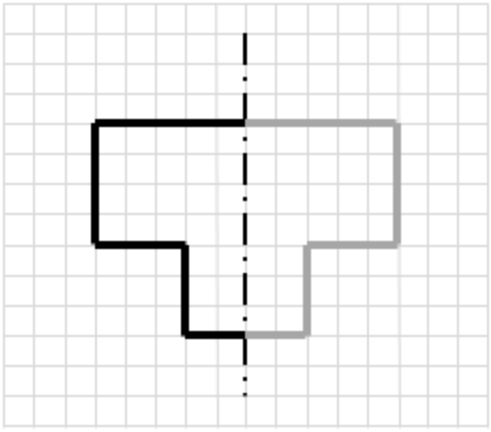


A'B'C' is a reflection of ABC.

- Point A' is a reflection of point A. They are both 2 squares away from the line of reflection
- Point B' is a reflection of point B. They are both 6 squares away from the line of reflection
- Point C' is a reflection of point C. They are both 1 square away from the line of reflection



The figure below is symmetrical, so reflecting one half will complete the figure:

		 Based on these examples, ask the learners to try to find some patterns (i.e. generalize) and list some properties of a geometric reflection. Here are some properties, make sure to encourage learners to say the same in different ways: - If you were to fold the diagram along the line of reflection, the reflected shape would exactly fit over the original shape. - Each point in the shape is the same perpendicular distance from the line of reflection as the corresponding point in the object - The reflected shape the same size as the original shape
	30 mins	In Day 2 worksheet, complete the reflections in Question 1 and Question 2 . Share and explain your solutions to a parent or adult or older sibling The parent should give feedback based on the following Assessment criteria: - The learner uses a mix of imagination, and physical means to verify responses (by placing the sheet against an actual mirror to see how the reflected shape looks) - The ability to explain or defend their answers - Learners will revise their solution in case they got something wrong.
	20 mins	**Rotation** Another kind of transformation is rotation. A geometrical rotation is a transformation that turns a figure or shape around a **given point** called the centre of rotation. Let's start with a simple activity: - Stand up facing one side of the room. - While standing in the same position, make one full rotation to go back to the initial position. - What angle have you covered in this rotation? (it is 360 degrees, or a full circle). - Now make a 180 degree rotation. This is a half circle, it makes you face the opposite direction in the room. - How about a 90 degree rotation?

If you are facing East (where the sun rises), a 90 degree rotation can make you either facing North or South.

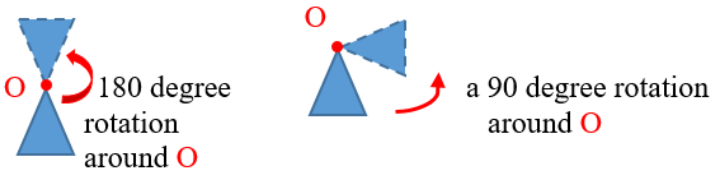
(Optional: In order to make a general agreement, mathematicians agree that a rotation is usually done counter clockwise, that is opposite to the clock rotation movement. So, if you are facing east, a rotation of 90 degrees will make you face North. Rotating from East to South is a 270 degree rotation).

- An example of Rotation is the difference between when a door is closed versus when it is open.

A rotation is defined by: a centre of rotation, and an angle.

- Stand facing one wall in the room, and rotate by 90 degrees. Where are you facing now?

- See the diagram below 2 examples of rotation: The centre of rotation must be defined.



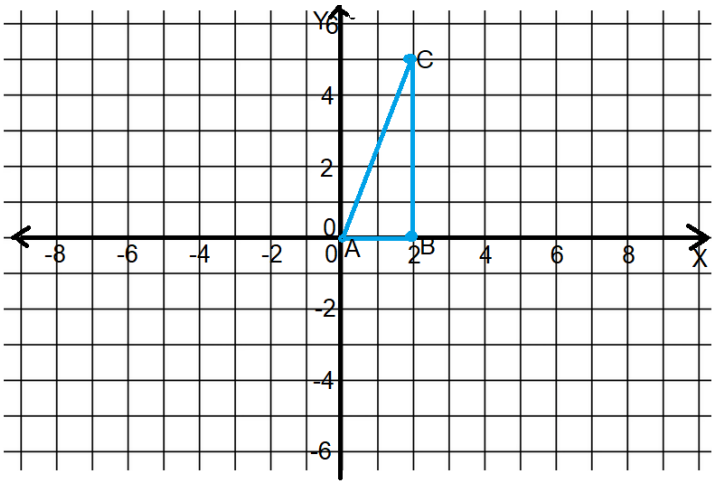
Properties of a geometric rotation

The object and the image of a rotated object have some mathematical properties

1. A rotation maps a line to a line, a segment to a segment and an angle to an angle
2. A rotation preserves lengths of segments
3. A rotation preserves measurement of angles

Example of a rotation.

Triangle ABC has the following vertices A(0,0), B(2,0) and C(2,5)

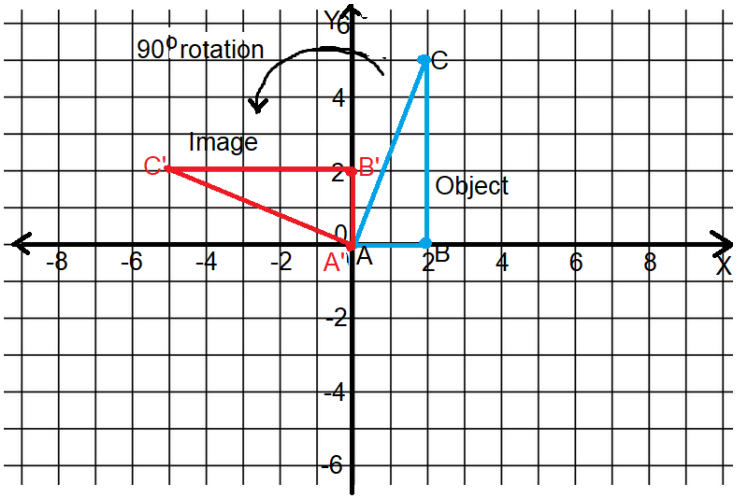


(i) Rotate the triangle ABC 90° counterclockwise about the origin (0,0). Draw the image A'B'C' of the rotated triangle on the grid. What are the vertices of the image A'B'C'?

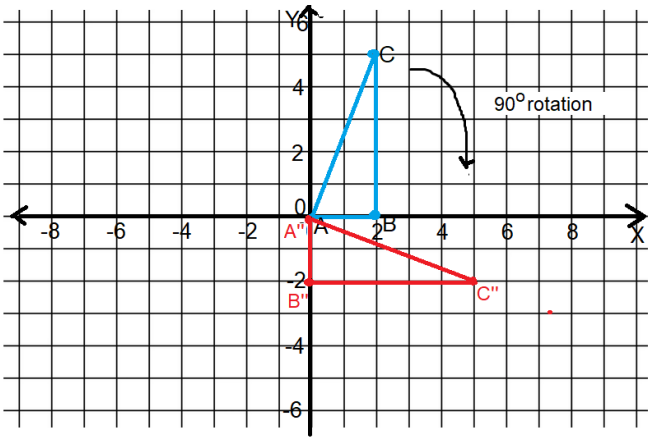
(ii) Rotate the triangle ABC 90o clockwise about the origin (0,0). Draw the image A''B''C'' of the rotated triangle on the grid. What are the vertices of the image A'B'C'.

Solution

(i) Triangle ABC rotated 90° counterclockwise about the origin point (0,0) as the centre of rotation



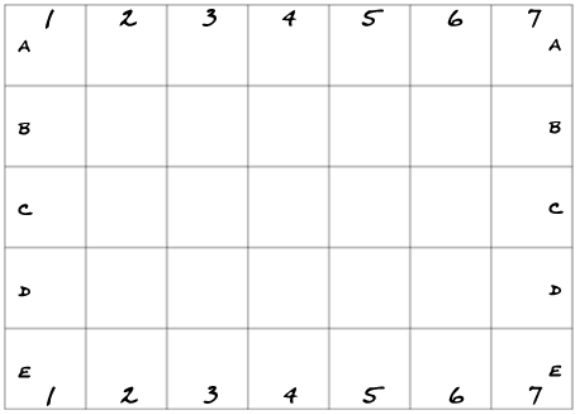
The image has the following vertices A'(0,0). B'(0,2) and C'(-5,2)

		(ii) Triangle ABC rotated 90° clockwise about the origin point (0,0) as the centre of rotation  The image A"B"C" has the following vertices A"(0,0), B"(0,-2) and C"(5, -2)
	20 mins	Solve number 3 and 4 on the Day 2 worksheet, and then explain your solution to a parent or adult. Assessment criteria: - The learner uses a mix of imagination, and physical means to verify responses (by putting the pencil on the Origin of rotation, and turn the whole sheet around the origin shows where the rotated shape will be) - Depth of understanding through the confidence in explaining and defending their answer
	15 mins	**Reflection** Learners will reflect on their own learning Reflection questions: - Describe Reflection in your own words - What does reflection change in an object? (probes: size, color?) - Describe Rotation in your own words - What does rotation change in an object? (probes: color, size, weight?) - What 3 important things have you learned about reflection - What 3 important things have you learned about rotation? - What else would you like to learn about reflection and rotation?
3	10 mins	Today we will use grids and some reflections to enhance our drawing skills. Grid Method of Drawing

When drawing anything, you need to get the proportion right. One way of achieving this is to aid your drawing with the use of a grid method of drawing.

The grid method of drawing involves placing a grid of squares over a reference photo and placing lightly an identical grid of squares on your drawing paper

Example:



Source: https://www.art-is-fun.com/grid-method

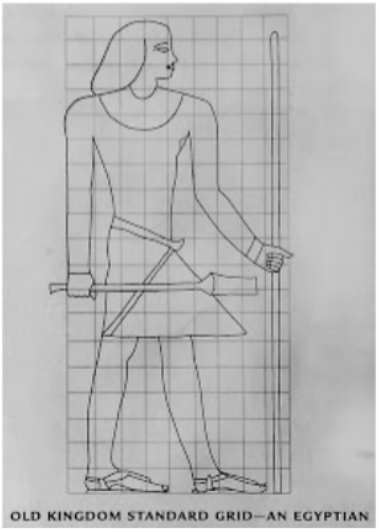
Remember when drawing the grids, the grids must have a 1:1 ratio. A 1:1 ratio means that you will have the exact same number of lines on your paper as you have on the reference photo and that the grid lines must be equally spaced – perfect squares.

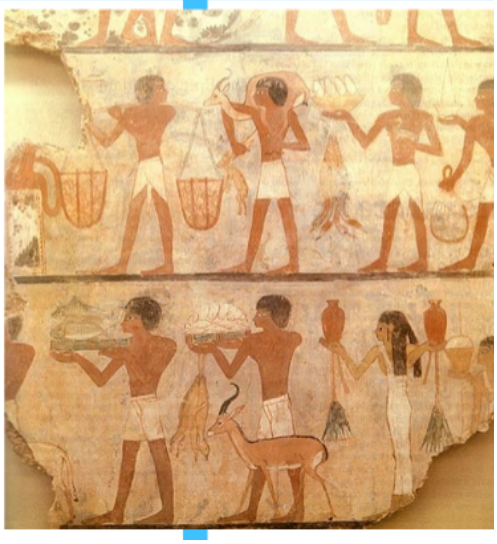
You then draw your subject on one square at a time and replicating what you see in each square

Once you're finished, you simply erase or paint over the grid lines on your paper and start working on your painting which will now be in perfect proportion

Many artists have used grids in drawing throughout history, see the below examples from ancient Egyptians (more than 4000 years ago), or Renaissance artists (more than 500 years ago).

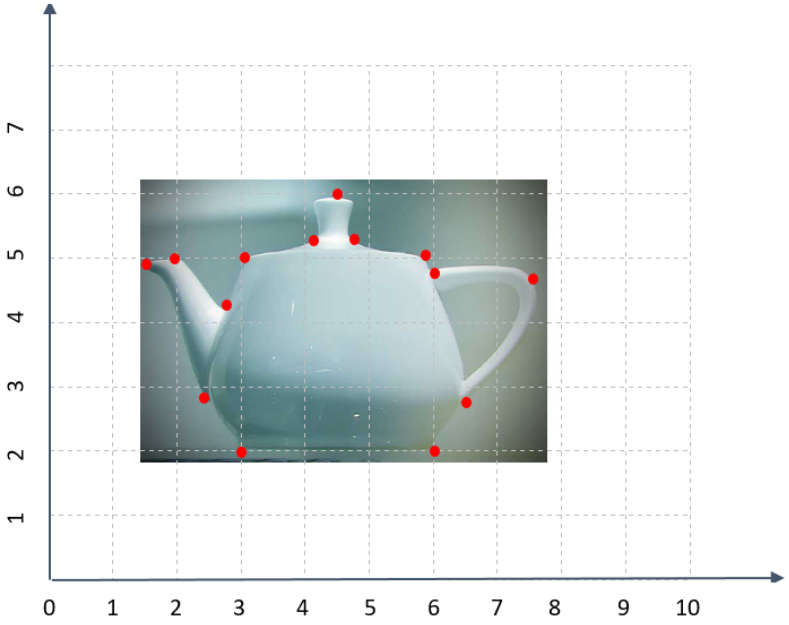
The ancient Egyptians at the beginning of the Middle Kingdom (around 2055 BC - 1650 BC) used grids by snapping a string soaked in red dye against their drawing surface to create grid lines. Their figures had very specific proportions that didn't change for centuries!

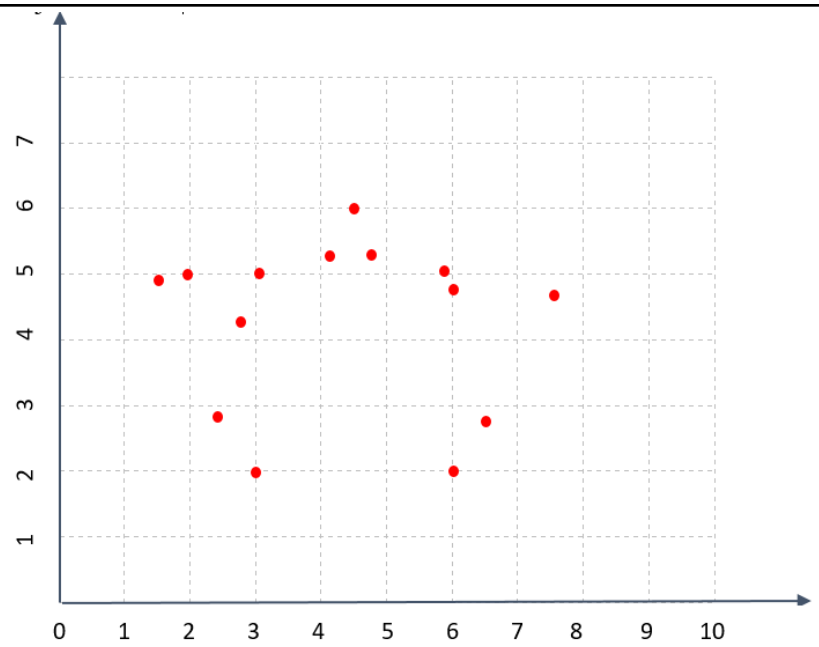




Renaissance artists would build a wooden frame, hammer in nails spaced equally apart and tie lengths of string from one end to the other to create their rows and columns. They would then place the wooden frame where they could look through the grid and see their subject from their workspace.

Source: https://prezi.com/eijrwmwhvgrg/grid-drawing-for-6th-grade/

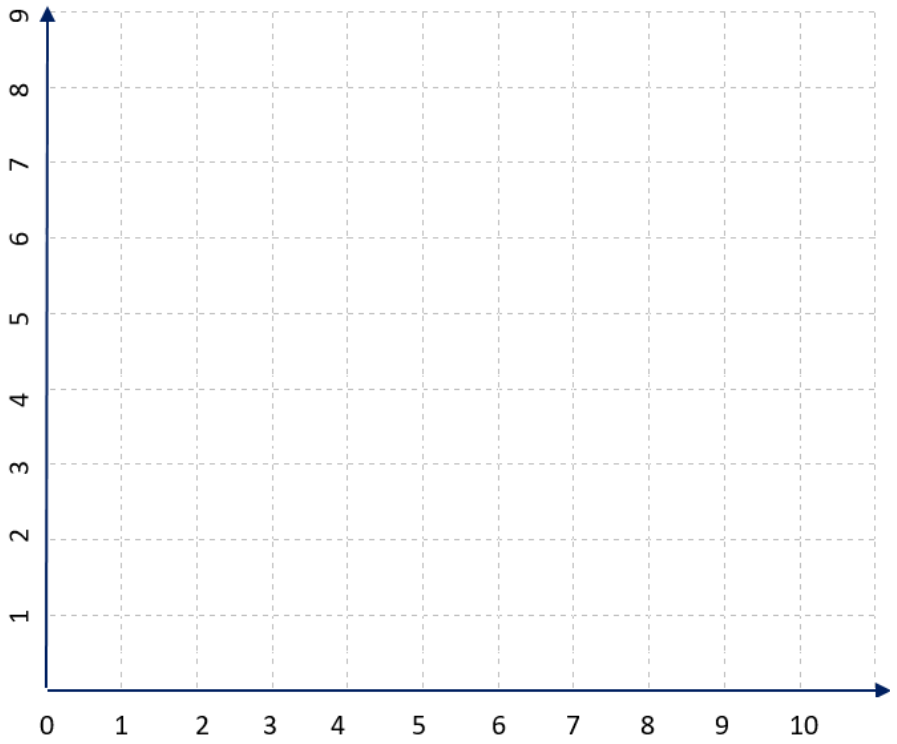
	30 mins	To what extent do you think this will help you improve your drawings? Let's start with an example: see this image of a teapot (called the UTAH teapot)  First try to draw this teapot on a piece of paper. (The learner takes 5-10 minutes to draw the teapot). Now let's use the grid technique and try again. A. Place the teapot through a grid, and mark the main points of the shape (see demonstration below).  Now try to draw the pot again using the dots.

		
	5 mins	Learners will reflect on their own learning. Questions to reflect on: ● To what extent has the grid technique improved your drawing? ● Do you think it is easier to draw using a grid? ● What objects would you like to try drawing using this grid technique?
	20 mins	Solve problem 1 on the Day 3 worksheet
	20 mins	Solve problem 2 on the Day 3 worksheet
	30 mins	There are some objects that we call 'Symmetrical'. This means that one half of the object is a mirror image of the other half. ● Is a circle symmetrical? ● What about a square? How about the human body? Try to use the grid technique and Reflections to draw the ladybird image in Question 3 of Day 3 worksheet.
	5 mins	Hang the drawings you completed today on a wall in the house.
Assessment Criteria:		- The accuracy of drawings compared to the original images - The rate of progress in terms of becoming faster at using the techniques - Use of properties of geometrical reflections in drawing reflection images - Use of properties of geometrical reflections in drawing reflection images - Accurate location of points on a coordinate grid

Additional enrichment activities:	-Make another copy of the ladybird drawing, and cut it into square pieces, then challenge your family members to arrange the puzzle. -Learners are challenged to choose a symmetric shape from the house, trace it into a grid, and make 2 copies of it. Then cut the 2 drawings into smaller square puzzle pieces, and arrange a competition amongst 2 groups of family members to complete the puzzle in a shorter period of time. - Learners use a grid to draw an image or an object of their choice. Once done, they may display it on a wall in the house.
Modifications to simplify the project tasks if need be	For a simpler version, learners may skip Question 2 on day 2 and problem 3 of day 3.

DAY 1 WORKSHEET

1. On the grid below, plot the points A(1,1), B(4,1), C(4,4) and D(1,4).



   a. What is the shape ABCD? Connect the points A to B, B to C, C to D, and D to A.

   b. Plot the points E(2,5), F(4,5), G(4,7). Connect the points E to F, F to G, and G to E. What is the shape EFG?

   c. Plot the points H(5,5), I(10,5), & J(10,7). Connect the dots H to I, I to J, and J to I. What is the shape HIJ?

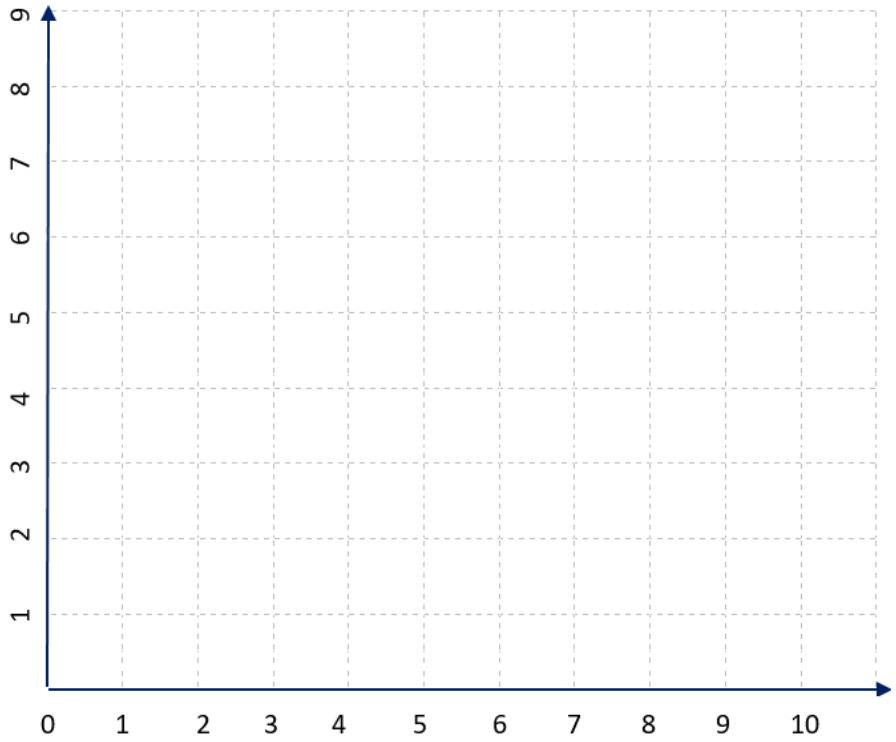
2. a. Find the area of the shape ABCD, by counting the number of area units, or grid squares, it encloses.

   b. Find the area of shape EFG. (hint: the answer is whole number).

   c. Can you find the area of shape HIJ?
   Probably it will be easier if you add a point K(5,7). HIJK is a rectangle.

First count the number of area units inside HIJK;
Then the area of HIJ is half of that.

3. On the grid below

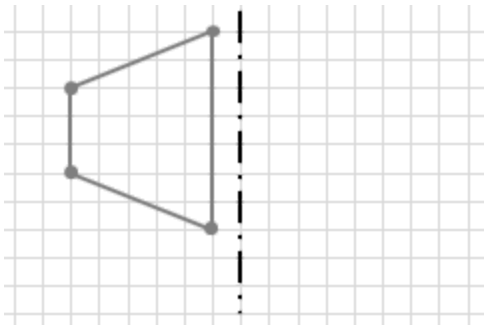


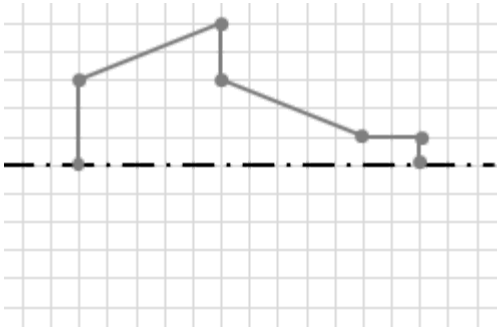
a. Draw the points A(3,2), B(9,2), C(9,7), D(7,5) and E(3,9).
b. Connect the points: A to B, B to C, C to D, D to E, and E to A.
c. Can you calculate the area of shape ABCDE ?

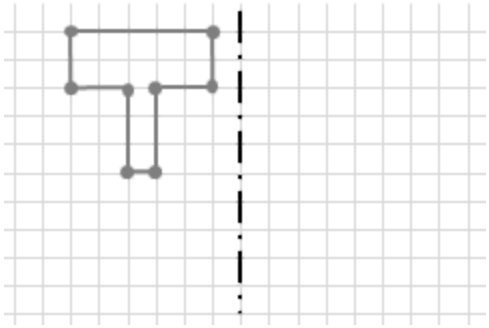
Hint: Add two points: F(9,5), and G(3,5). The area of ABCDE can be found by adding the areas of 3 shapes:
Area of rectangle ABFG + Area of triangle CDF + Area of triangle DEG.

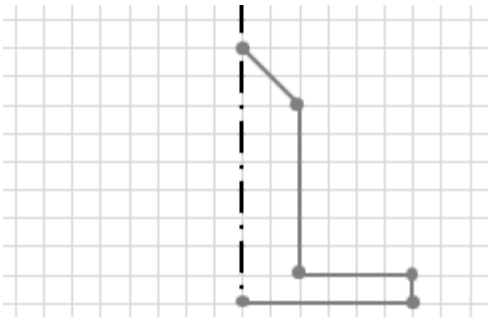
DAY 2 WORKSHEET

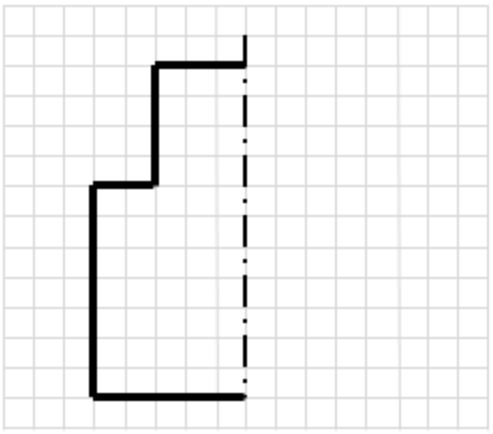
1. Draw the reflection of the shapes using the dashed line as Axis of Reflection (source: https://www.helpingwithmath.com/printables/worksheets/geometry/4g3-symmetry02.htm)
   Hint: The reflection of each point will be the same number of squares to the other side of the mirror line.

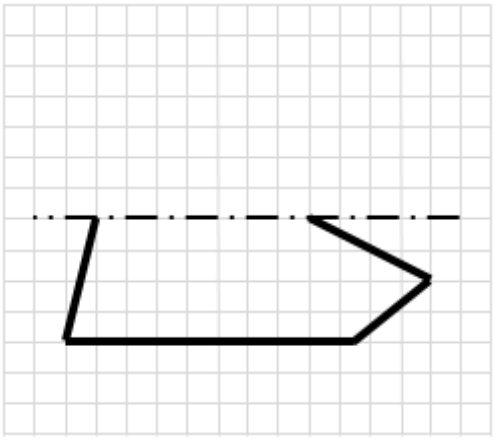




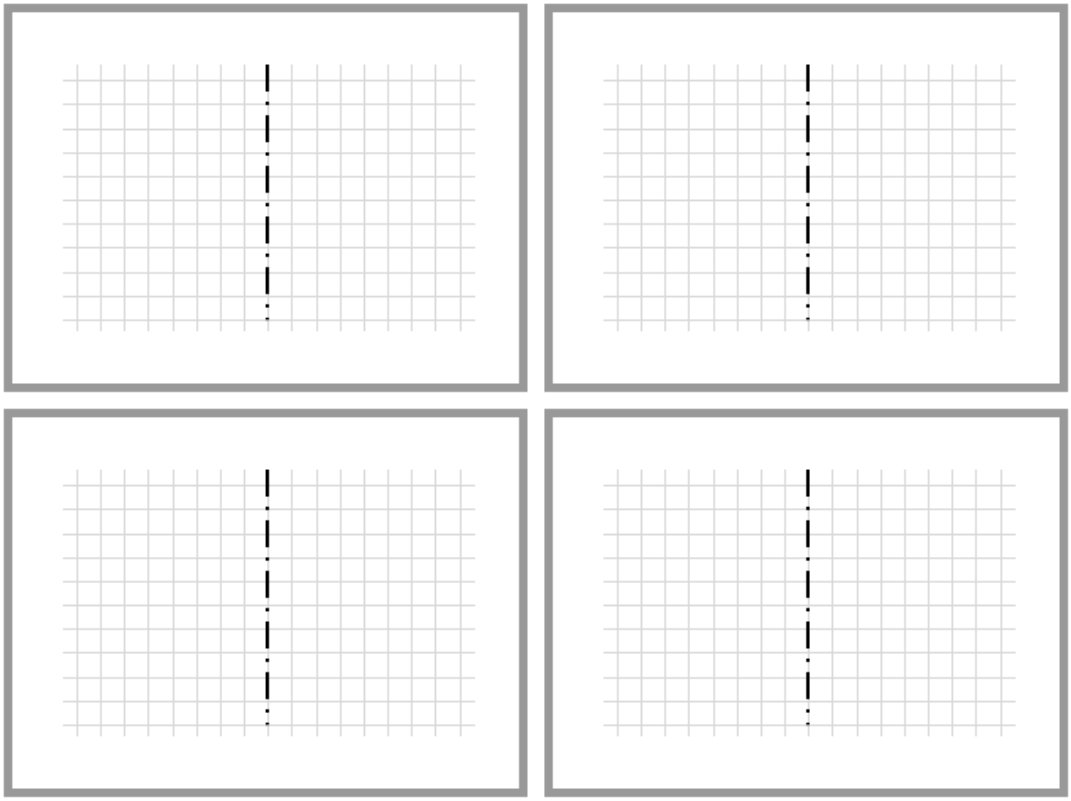




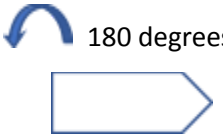




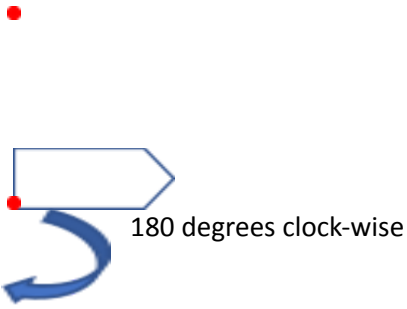
2. Draw any figure or shape you like and then draw the reflection

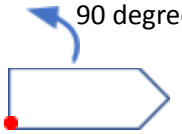


3. Draw the rotated shape around the origin (red dot), and with the respective angle:



180 degrees counter clock-wise





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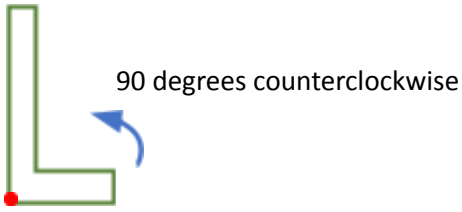
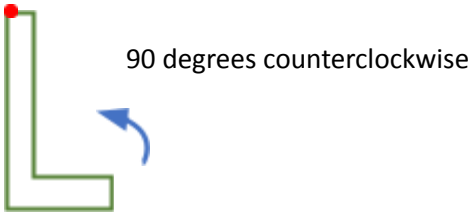
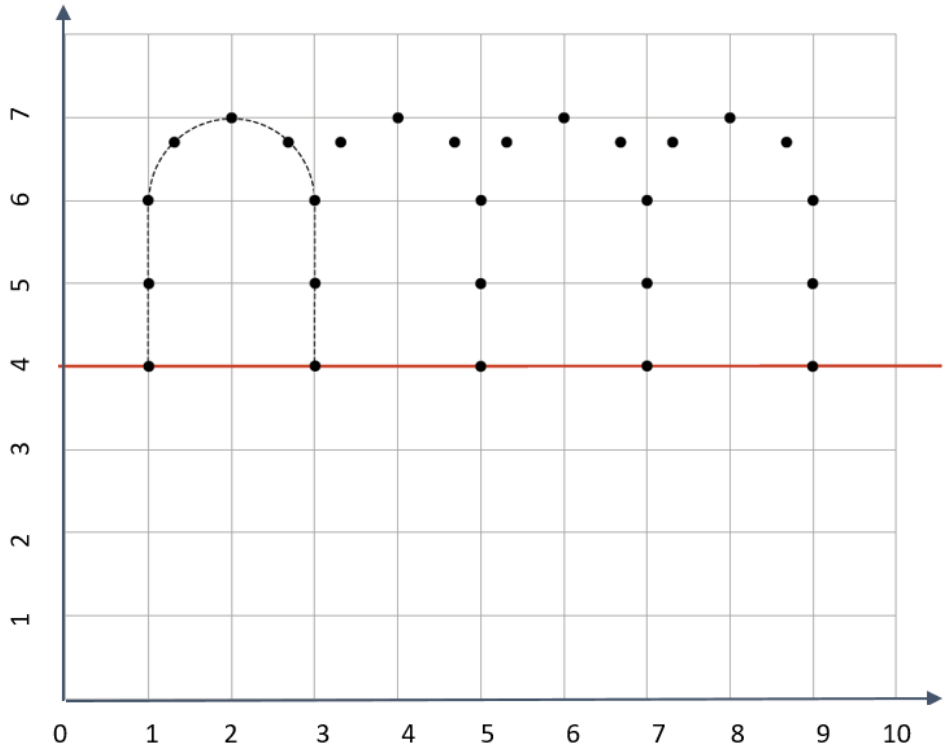




DAY 3 WORKSHEET

1. Here is a picture from the backyard of the Museum of Islamic Art in Qatar.

a. Using the grid and points below, try to trace the arches.



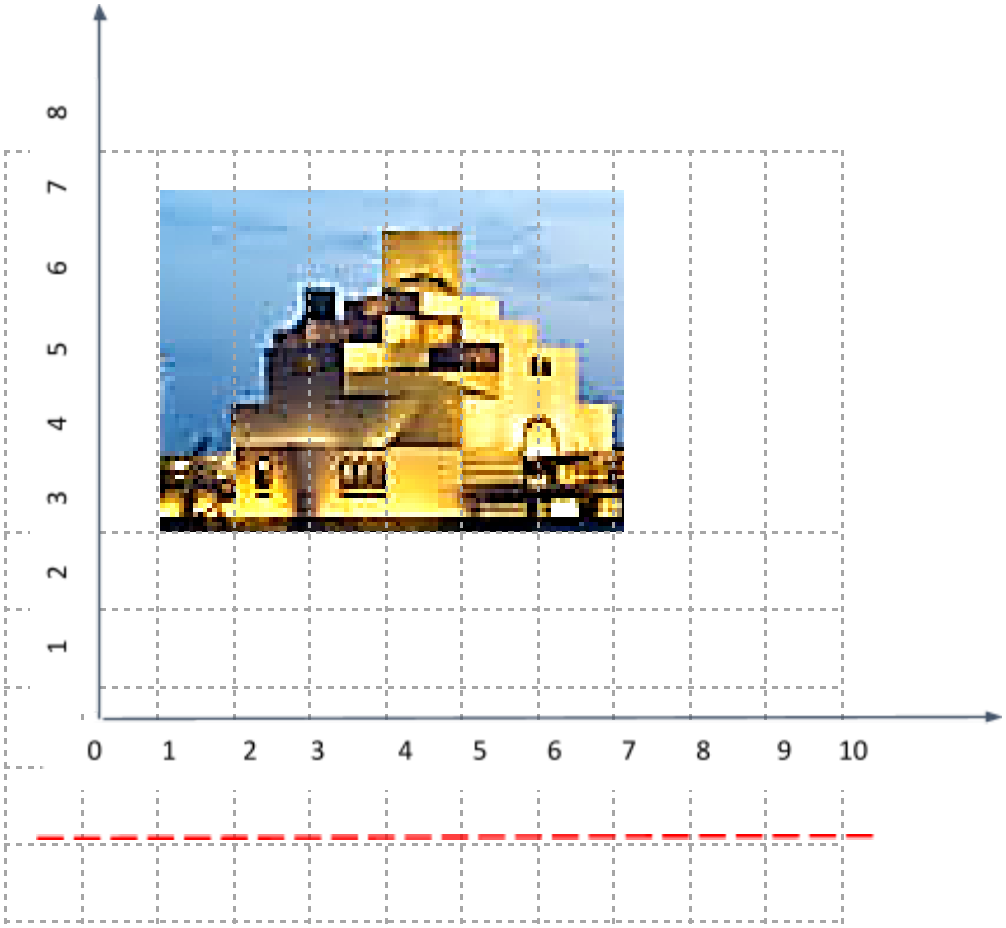
The mirror line, or

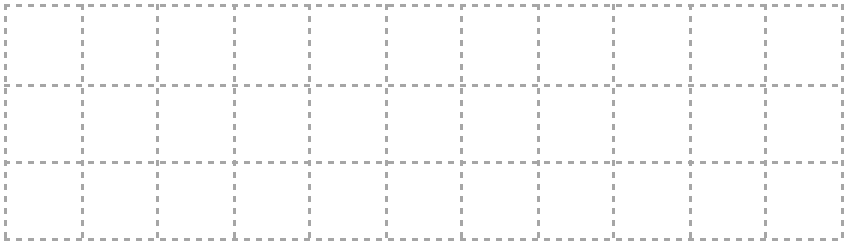
Axis of Reflection

b. After drawing the arches, use the red line as an axis of reflection, and draw the reflection of the arches.

2.

Here is another picture of the Museum of Islamic Art with its reflection in the water. Use the grid below to draw the reflection of the museum.

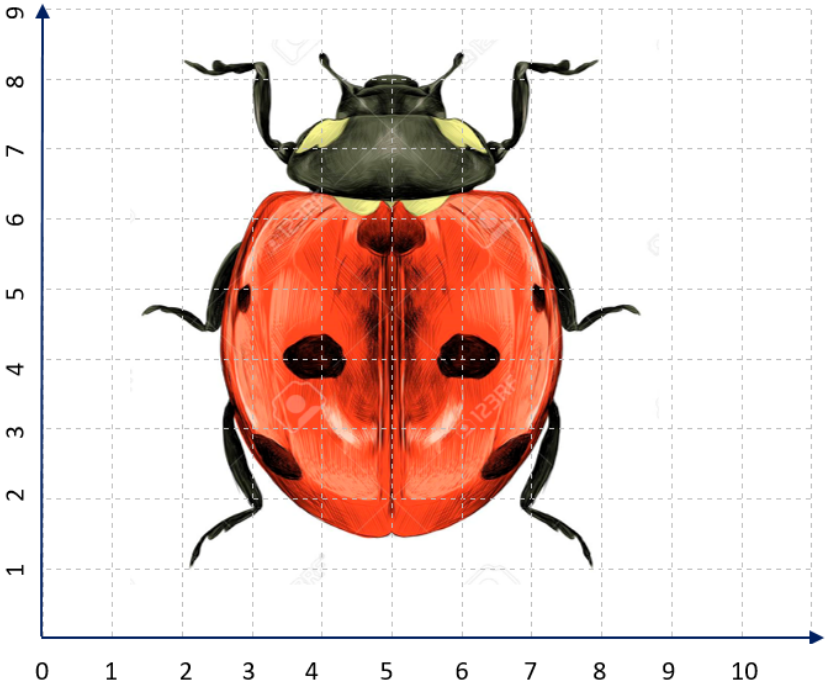




Axis of reflection

Hint: first mark the main corner points of the museum structure, then make the reflection of each point, and lastly connect them to get the trace of the museum building's reflection.

3. Notice the ladybird image below. You are challenged to recreate this image on the grid below. Hint: the Ladybird's body is symmetrical, which means that one side of it is a reflection of the other side- see the dashed line below is the axis of symmetry.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